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CALCULATION OF THE UNSTEADY THERMAL STATE OF A SLAB HEATED BY A MOVING SOURCE

S. V. Osovets, E. V. Toropov,
A. V. Prokhorov, and V. L. Kirillov

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Using numerical simulation, the authors have performed calculations of the unsteady thermal state of a concrete slab with its surface heat-treated by a moving plasmatron. Temperature asymptotics are obtained for the surface heating zone and the technologically important heating zone.

We consider the problem of the nonstationary thermal field in an "infinite" heat-insulated slab in coordinates associated with a surface heat source of intensity $q(x)$. The initial equation and the boundary conditions are as follows:

$$\frac{\partial T}{\partial t} = V\frac{\partial T}{\partial x} + \chi\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right), \tag{1}$$

$$\left.\begin{aligned} &-\lambda \left.\frac{\partial T}{\partial t}\right|_{z=0} = q(x)\,, \\ &T(\infty, t) = 0\,, \\ &T(x, z, 0) = 0\,, \end{aligned}\right\} \tag{2}$$

where $q(x)$ is the specific heat flux from a normally distributed source.

In constructing a finite-difference analog and developing an algorithm for solution of this problem we face a number of special features associated with the necessity of using a nonuniform computational grid with respect to the space variables. We will characterize them in brief.

First, the need for introducing a nonuniform grid with respect to x and z arises because of the different scales of heat transfer in these directions. The characteristic dimension is (0–10) mm for z and (0–1) m for x. In our case it is convenient to use a quasi-uniform grid prescribed by the transformation

$$p = \alpha \tan\left(\frac{\pi l}{2}\right), \tag{3}$$

where α is the characteristic dimension (scale); $p \in (-\infty, +\infty)$; $l \in (-1, +1)$.

Second, since the mathematical model is constructed for an infinite slab, in using the finite-difference approximation it is important to account correctly for "infinite points." An overall view (schematic) of the quasi-uniform grid with respect to x and z is presented in Fig. 1.

Third, in calculations of this kind the conservative property of the difference grid is sometimes disturbed. Therefore, in such cases it is necessary to undertake special measures [1] or use a rather cumbersome integrointerpolation method.

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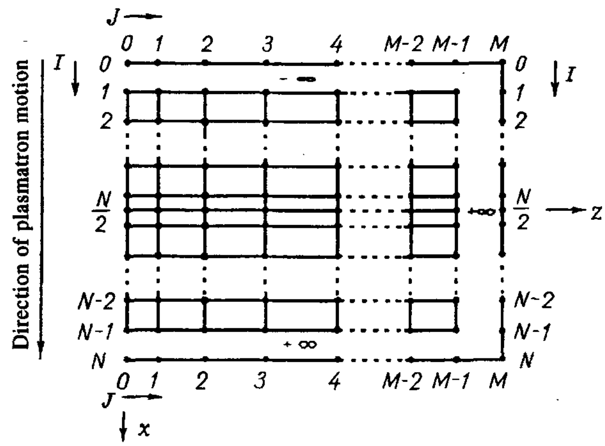
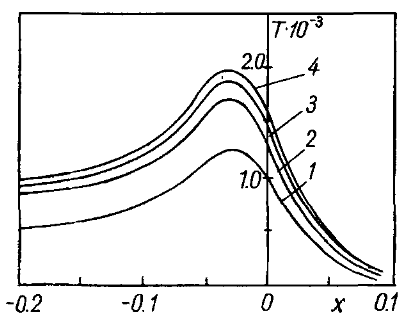


Fig. 1. Overall view of the quasi-uniform grid for the infinite slab.



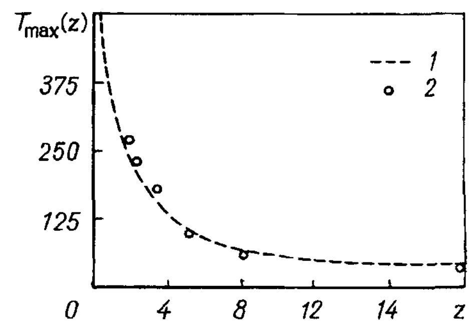


Fig. 2. Temperature field on the slab surface (calculation by the finite-difference model): 1) 0.1 sec; 2) 0.2; 3) 0.3; 4) 0.5. $T \cdot 10^{-3}$, °C; x, m.

Fig. 3. Comparison of calculated (1) and experimental (2) values of the maximum temperature in the slab. $T_{max}(z)$, °C; z, mm.

Nodal grids with respect to x and z are constructed with the aid of transformation (3). The characteristic dimension for z is $\alpha_z = 0.01$ m; for x it is $\alpha_x = 0.1$ m. As a whole, this scheme is undoubtedly stable with second order of accuracy in the space variables and first order of accuracy in the time $\sim O(h_x^2, h_z^2, h_\tau)$. By introducing the quasi-uniform grid the conservatism property of the scheme remains almost undisturbed.

Figure 2 shows results of calculation of the temperature on the slab surface at different times. It is seen that a state approaching heat saturation is attained on the surface in 0.3–0.5 sec. As the depth z increases, the time of establishing a temperature close to the maximum one increases substantially and, in particular, in the calculation of the maximum temperature at depth z (Fig. 3) it is 2–4 min for z = 5–8 mm. In Fig. 3 the calculated maximum temperatures $T_{max}(z)$ are also compared with experimental data of [2]. The agreement between theory and experiment is quite satisfactory. The calculations presented in Figs. 2 and 3 were performed for the following values of the parameters: $P = 50$ kW; $V = 0.3$ m/sec; $T_0 = 20$°C.

Figure 4 shows profiles of temperature fields in the concrete slab calculated by the finite-difference scheme for ($t \to \infty$) and by an approximate analytical model [3]. It is seen that with increase in the depth z the temperature curves decrease as the distance from the source increases but do not intersect, and the difference between the analytical and numerical calculations has a tendency to increase.

Now, using results calculated by the finite-difference model and employing the approach partially described in [4], we will consider in brief the derivation of simple temperature asymptotics that are useful for engineering calculations.

It is known that heat treatment of silicate materials by a powerful plasmatron source is accomplished, as a rule, at a high rate and is characterized by large Peclet numbers: $\mathrm{Pe} = Vd/\chi \sim 10^4$–$10^5$.

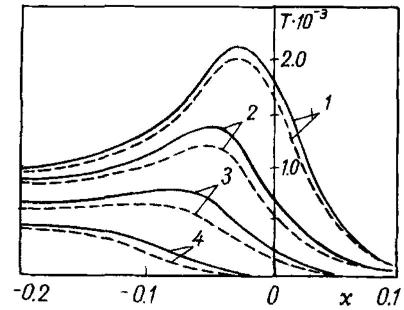


Fig. 4. Temperature field in the concrete slab: a comparison of the finite-difference (dashed curves) and analytical (solid curves) models (P = 50 kW; V = 0.3 m/sec; T_0 = 20°C): 1) z = 0; 2) 0.32; 3) 0.73; 4) 1.37.

At the same time due to the presence of substantial temperature drops along the normal to the surface, the cooling time in the surface layers turns out to be of the same order of magnitude as the heating time $\sim d/V$, which is substantially smaller than the time of cooling due to lateral heat removal $\sim d^2/\chi$.

With regard for the foregoing, the temperature field in the slab below the source can be conventionally subdivided into three zones. In the first zone, located in the immediate neighborhood of the surface (the near-surface layer), the temperature decreases more intensely and its distribution depends substantially on the distribution of the surface heat sources. In the middle zone (the technologically important layer) the dependence of the temperature on the distribution of the heat sources is partially "forgotten" and now the decisive factor is the linear dimension of the heat source in the direction of motion. In the most remote third zone the temperature field almost totally "forgets" the character of the surface heat source, and the latter can be represented in the form of a point source.

On the other hand, in theoretical studies and calculations of thermal processess use is made, as a rule, of the following types of heat sources: point, linearly distributed, and circular with a Gaussian distribution of the power density. In solving the problem under consideration on slab heating by a heat source uniformly and linearly moving over the surface, it becomes necessary to use all three types of heat sources to determine the asymptotic behavior of the maximum temperature as a function of the depth of heating z. We will consider this issue in more detail.

For a point heat source, as shown in [5], the maximum value of the temperature in the thermal cycle along the axis of source motion decreases in accordance with the asymptote $\sim 1/z^2$:

$$T_{\max}(z) = \frac{2Q\chi}{\pi\lambda V}\frac{1}{z^2}. \tag{4}$$

However, under conditions of high Pe numbers, use of this estimate for prediction, for instance, in the middle (technologically important) zone substantially distorts the dependence $T_{\max}(z)$. To solve this practically important problem, the authors of [6] found an asymptotic approximation for a linearly distributed point heat source:

$$T_{\max}(z) = \sqrt{\left(\frac{2}{\pi}\right)}\frac{q_l\chi}{V\lambda}\frac{1}{z}. \tag{5}$$

The authors showed that the dependence established by them agrees satisfactorily with an experiment on measurement of the maximum temperature as a function of the depth in the technologically important zone. For a concrete slab, the range of this zone according to their evaluation is $0.5 \le z < 150$ mm.

In the present work the asymptotic behavior of $T_{\max}(z)$ for the technologically important zone is obtained with account for the Gaussian character of the power density distribution of the surface heat source:

$$T_{\max}(z) = \sqrt{\left(\frac{2}{e}\right)}\frac{r_0 q_0\chi}{\lambda V}\frac{1}{z}. \tag{6}$$

TABLE 1. Maximum Temperature as a Function of the Depth ($V = 0.3$ m/sec; the source power is 50 kW)

T_{max}, °C	1818	1475	1134	790	485
z, mm	0.1	0.2	0.3	0.4	0.5

This asymptotics differs from the preceding one by ~30%. The results of calculation of $T_{max}(z)$ by the finite-difference model agree well with experiment (see Fig. 3).

It is also important to know the asymptotic behavior of the temperature in the near-surface layer. Results of calculation of the maximum temperature under the moving source for different z ($0 < z \leq 0.5$ mm) by the finite-difference model are presented in Table 1.

Approximation of the calculated data gives a linear temperature asymptotics for the surface layer:

$$T_{max}(z) = A - Bz, \tag{7}$$

where $A = 2200$ and $B = 3525$ are the parameters of the straight line.

The obtained asymptotic forms of the temperature will make it possible to solve the inverse heat-conduction problem: using the experimentally measured maximum temperature in the technologically important zone, recover the power and the characteristic dimension of the surface heat source and, possibly, the maximum temperature of the surface.

NOTATION

x, z, coordinates of a point in the slab, respectively, in the direction of plasmatron motion and along the depth of the slab; T, temperature; $T_{max}(z)$, maximum temperature over the depth z; V, velocity of the heat source; χ, thermal diffusivity; λ, thermal conductivity; P, power of the source; T_0, initial temperature of the slab; α, l, transformation parameters for constructing the quasi-uniform grid p; t, time; I, N, index of the grid and its maximum value in the x direction; J, M, index of the grid and its maximum value in the z direction; h_x, h_z, h_τ, grid steps with respect to the space and time variables; Pe, Peclet number; d, thickness of the slab layer; q_0, specific heat flux on the axis of the Gaussian heat source; r_0, characteristic dimension of the Gaussian heat source; q_L, specific heat flux of the linearly distributed point heat source; Q, integral power of the source; e, base of natural logarithms; A, B, parameters of the linear temperature asymptotics.

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